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ANALYSIS AND IMPROVEMENT OF ITERATION METHODS FOR SOLVING AUTOMATIC CONTROL EQUATIONS

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This report covers work done during the first six calendar months of this grant. During this time labor was charged for three months at full time and four months at one-third time for a total of four and one-third man months. A portion of this total time was spent on duties as chairman of the Program Committee of the 1965 Joint Automatic Control Conference. Time was also taken to prepare a short paper, "A New Array for Application of Routh-Hurwitz Stability Criterion." This has been accepted for publication by Control Engineering Magazine.

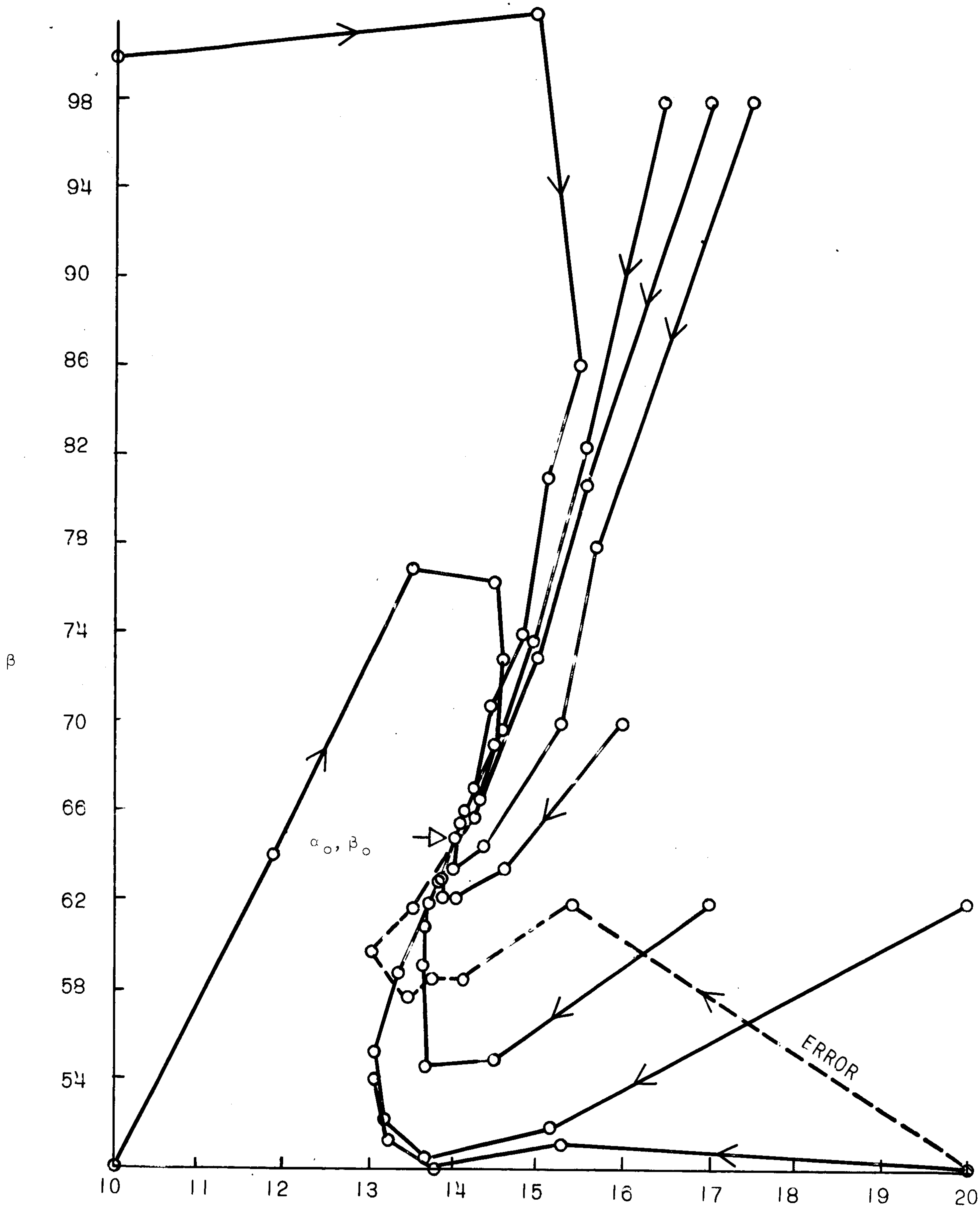
Research on improving iteration methods has centered primarily on convergence of the methods for equations having all complex roots. It is felt that previous work on equations having all real roots and on those having both real and complex roots has resulted in methods which are quite adequate. On the over all problem it has seemed advantageous to pursue several avenues. These are:

1. Numerical solutions of a large number of fourth and higher order equations having roots in various parts of the complex plane.
2. The seeking of an analytic expression defining a region of convergence or divergence about known roots.
3. The study of root movement or sensitivity by the method of plotting root loci.

Item (1) has been most informative from the standpoint of understanding behavior of the iteration in both convergent and divergent regions. It has shown that the region of convergence may be of somewhat irregular shape and may be extremely small or may cover the entire area of possible root locations. A similar statement holds for roots for which the iteration is non convergent.

In an attempt to better visualize the form of iterative steps taken, these steps have been plotted for numerous starting points for equations in which no extrapolation has been used. An example is shown in Fig. 1 for the equation

$$x^4 + 20\,x^3 + 162\,x^2 + 572\,x + 845 = 0 \qquad (1)$$



98
94
90
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74
70
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58
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β
α_o, β_o
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α

FIGURE I

Starting points were picked which represented all reasonable possible root locations. The axes represent the coefficients of the trial quadratic.

$$x^2 + \alpha x + \beta \tag{2}$$

Each indicated trial is marked by small circles and successive iteration points are connected by straight lines. The iteration proceeded as shown by the arrows. Note the strong tendency of the iterations to swing around and approach the root location (represented by α_0 and β_0 in the quadratic) along a common and fairly straight line. One line is shown for an iteration having a numerical error in the early stages. This line then corrects itself and swings around as do the others.

Repetition of the above scheme for many equations reveals the spiraling tendency of the iteration values. It also reveals that the spirals for non convergent iterations may approach a curve about the actual root location which the iterations will not cross. Indeed, trial values picked within the region result in divergent iterations which appear to approach the same curve from the inside.

The "curves" plotted in Fig. 1 bear a marked resemblance to phase trajectories used in the study of non-linear dynamical systems. This has led to the attempt, Item 2, to study root convergence by some of the techniques used for non-linear systems, i.e., identification of limit cycles and the use of Lyapunov's Direct Method. As an aid here references [1] and [2] have been studied extensively. No useful results have been obtained as yet but this is the present area of endeavor.

Another method of studying convergence is given in Item 3. This is the method of root loci. This has added additional insight in the iteration studies but has also opened two new areas which appear to be of some importance. These are reported briefly below.

The first area involves what might be called moving roots in the complex plane. If we are given an open loop control system transfer function of the form (using Laplace notation),

$$G(s) = \frac{K\,(s + a)\,(s + b)}{s\,(s + c)(s + d)} \tag{3}$$

we can plot a locus of closed loop roots with K as a variable. This, of course, is well known. We can also hold **K** constant and plot a locus versus any of the other parameters, a, b, c and d.[3]. This last seems to offer an excellent method of both studying and designing systems and controls which have time varying parameters. For example, consider a system to be controlled having the transfer function

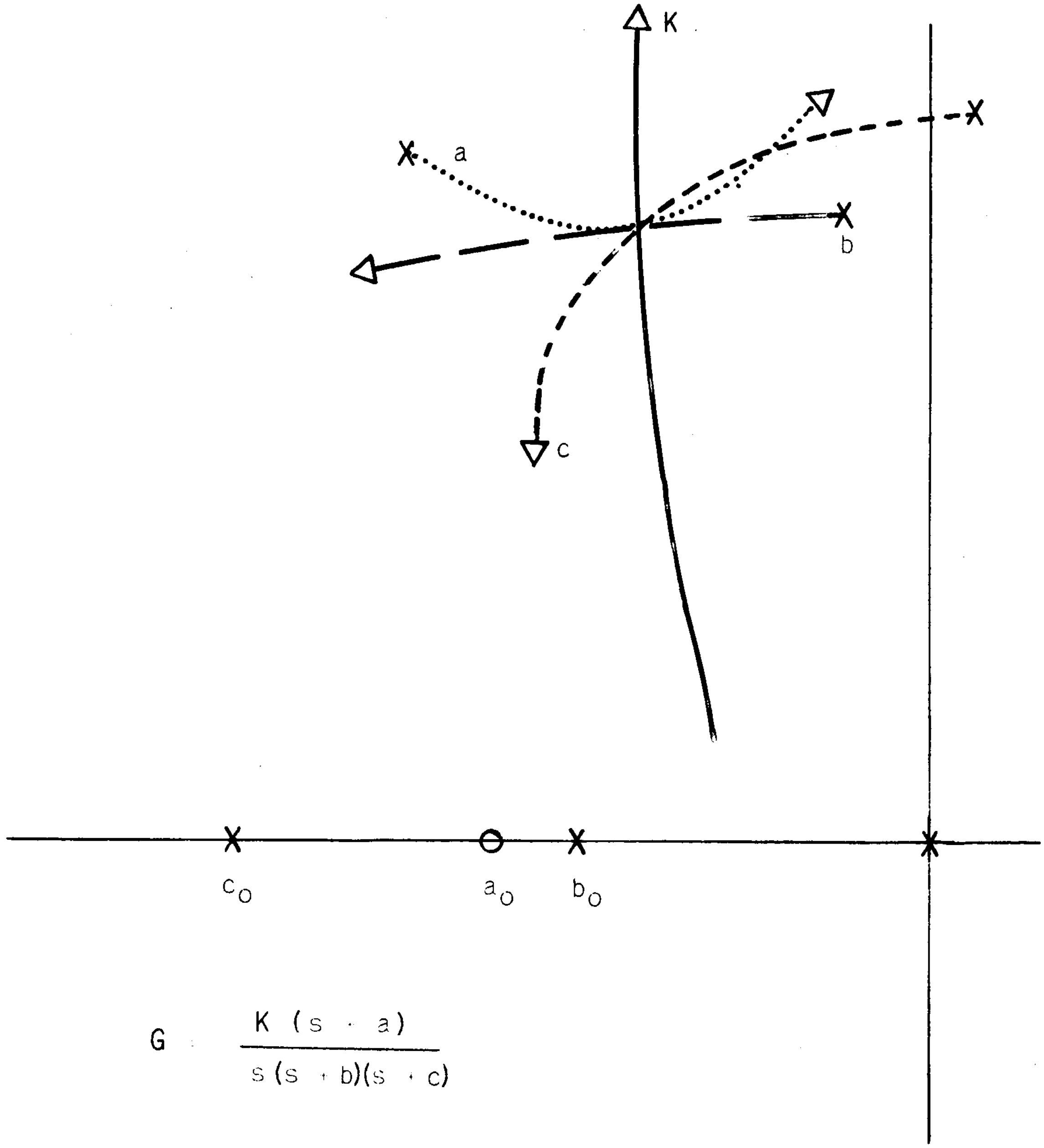
$$G_1(s) = \frac{K}{s(s+b)} \tag{4}$$

where b may vary from some initial value to larger values. Now add series compensation in the form

$$G_2(s) = \frac{s+a}{s+c} \tag{5}$$

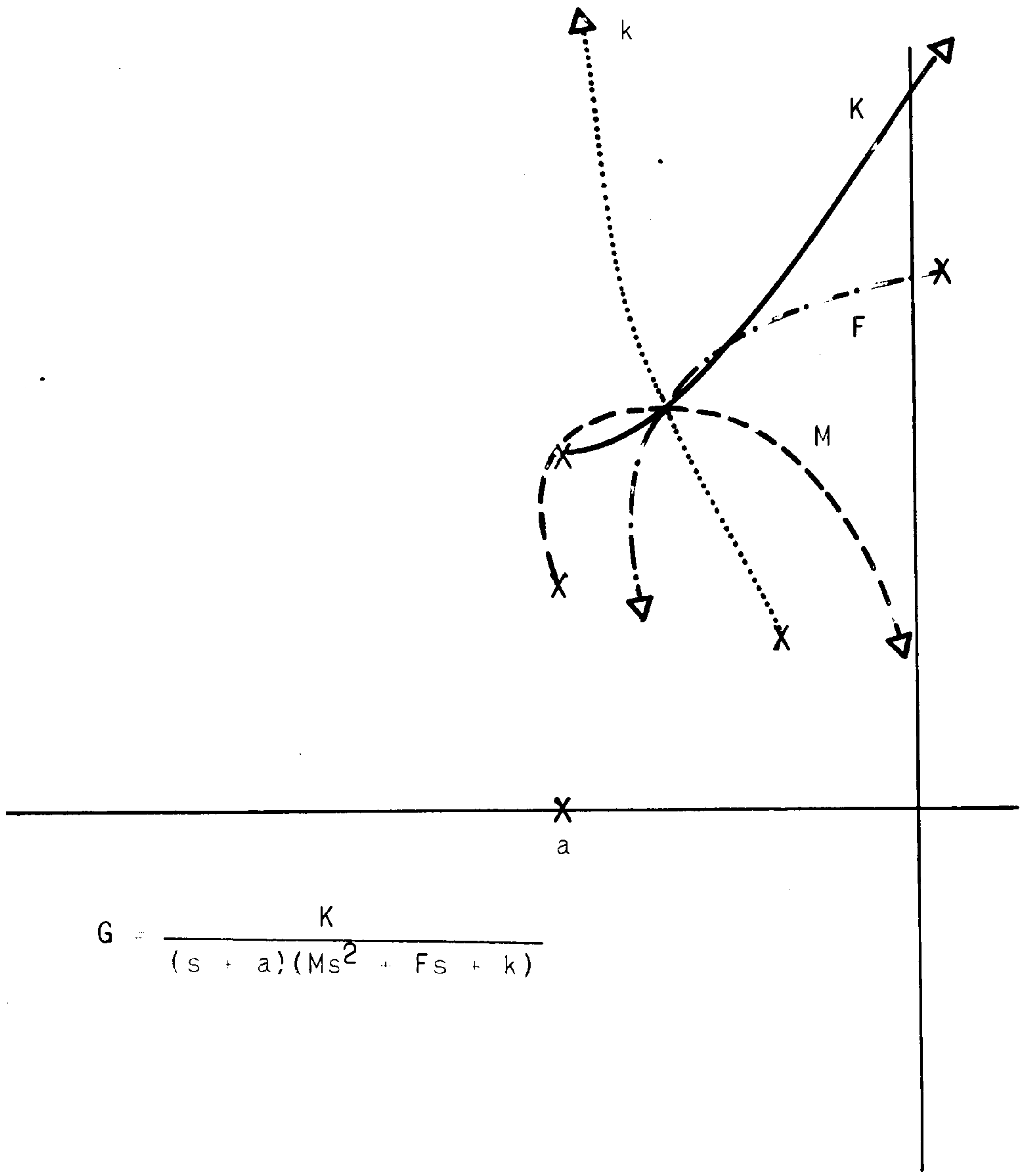
Loci for K, a, b, and c are plotted in Fig. 2. The point where all the loci cross represents the closed loop roots for the initial values of K, a, b, and c. Each loci then represents the motion of the closed roots for a variation of that particular parameter. In any realistic system the total variation in b would be limited. Thus a form of adaptive controller can be constructed based on this motion. If b increases, for example, a may be increased or c decreased to keep the closed loop roots in very nearly the same spot. This method may also be used for systems with open loop transfer functions as shown in Fig. 3.

Considering Equations (4) and (5), we also appear to have a design scheme. Suppose, in Equation (5), that a = b. The open loop transfer function is then given by Equation (4) with the controller having no effect. We may, however, still plot loci of a and b even though they are equal. Such an example is shown in Fig. 4. This offers a design method. A control function with a = b (or possibly several) can be put in the neighborhood of the system poles. Then, once the system closed loop roots are located, they can be moved to a desirable location along one of the a and/or b loci.



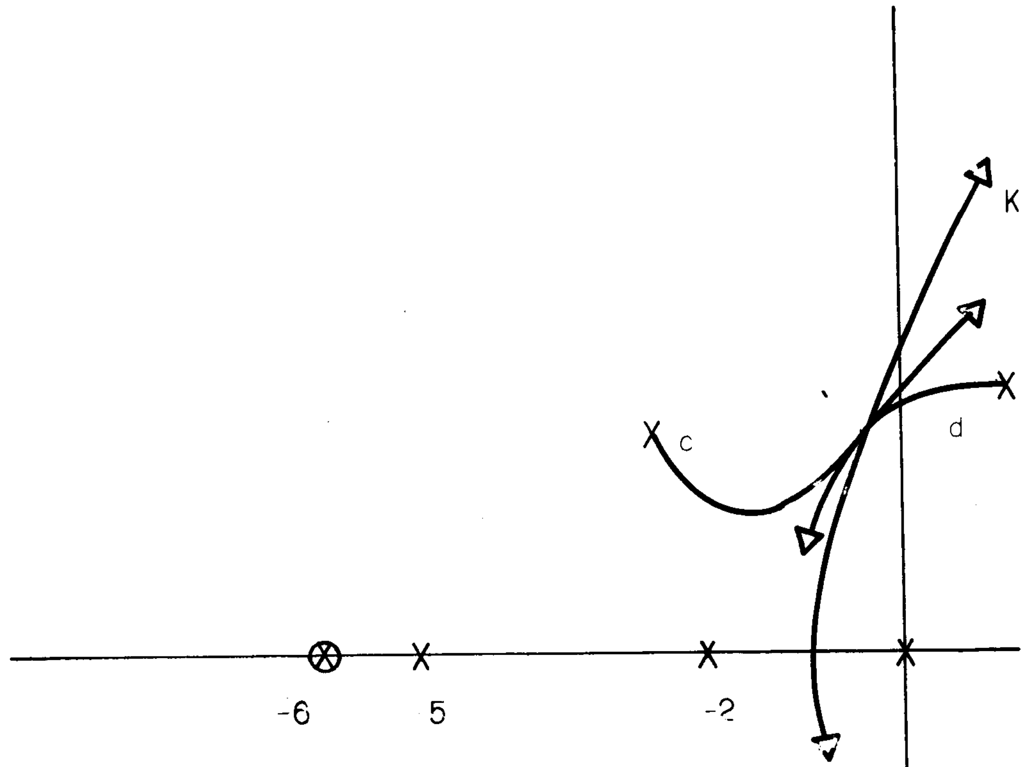
$$G = \frac{K\,(s + a)}{s\,(s + b)(s + c)}$$

FIGURE 2



$$G = \frac{K}{(s + a)(Ms^2 + Fs + k)}$$

FIGURE 3



$$G = \frac{K(s+c)}{s(s+2)(s+d)(s+5)}$$

$c_0 = d_0 = -6$

FIGURE 4

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Summarizing, present efforts are in the area of Item 2, the search for analytic expressions defining areas of convergence. Item 3 with the results reported above appears to offer promise of a new area of research. This will probably be pursued at a later date in the form of another proposal.

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